Pure Appl. Geophys. 170 (2013), 2013–2015

DOI 10.1007/s00024-013-0656-z

Pure and Applied Geophysics

Book Review

Mathematical Modeling of Earth's Dynamical Systems: A Primer by Rudy Slingerland and Lee Kump, Princeton University Press, 2011; ISBN: 978-0-691-14513-6 (Hardcover), ISBN: 978-0-691-14514-3 (paperback)

Andrzej Icha[1]

The aim of mathematical modeling is to obtain in quantitative form information regarding the selected aspects of reality by use of the mathematical concepts and apparatus.

The concise textbook: "Mathematical Modeling of Earth's Dynamical Systems: A Primer" by R. Slingerland and L. Kump, provides the mathematical and computational background for earth scientists and students who are less familiar with the role of mathematical models in the geosciences, especially in the description of the Earth's surface processes.

The book starts with Chapter 1, where in the subsequent sections the authors present the basic philosophy of mathematical modeling. Overviewed are standard notions, definitions and steps in model building and solving. Two examples are described briefly: simulation of the Chicxulub Impact and the storm surge of hurricane Ivan in Escambia Bay. The last several subsections deal with well-posed differential dynamical models in the Hadamard sense.

Chapter 2 deals with the numerical analysis background for a finite difference approach to solve partial differential equations. Some elements of matrix algebra are reviewed first. Convergence, stability, and consistency of finite difference schemes in the solution of a model, one-dimensional (1-D) diffusion equation are explained and discussed briefly.

Chapter 3 is devoted to the box (or "toy") modeling. Toy models in geosciences are developed to make simpler the description of complex earth systems while preserving only essential attributes of key components and processes. The methodology of box modeling is presented through a few examples including: radiocarbon balance of the biosphere (as a one-box model); the carbon cycle (as a multibox model); 1-D energy balance climate model and Rothman Ocean (i.e., two reservoirs model of the Proterozoic Era ocean carbon cycle).

Mathematical description of toy models leads to a system of ordinary differential equations with a prescribed set of initial conditions. As indicated in Chapter 2, the standard approach is to convert these equations to algebraic equations using finite differences and then use methods from linear algebra to solve the equations. Included are: the forward Euler method, predictor–corrector methods and the backward Euler method. Some remarks regarding the model enhancements are presented also.

Chapter 4 discusses the 1-D diffusion equation, a paradigm for many of the physical problems in which a conservative property moves through space at a rate proportional to some gradient. Three examples are explored and analysed to illustrate the earth phenomena which can be concisely described by such an equation, namely: dissolved species in a homogeneous aquifer, dynamics of a sandy coastline and diffusion of momentum. Exact solutions to diffusion problems are compared with numerical calculations by utilization of the Crank–Nicolson scheme.

Chapter 5 continues the study of diffusion problems. The analysis is extended to two dimensions (2-D) and three model issues are considered: diffusional landscape, pollutant transport in a confined aquifer and the thermal effects of radioactive waste disposal.

[1] Pomeranian Academy in Słupsk, Institute of Mathematics, ul. Arciszewskiego 22d, 76-200 Słupsk, Poland. E-mail: majorana38@gmail.com

With general initial-boundary conditions and the complex geometry, the suitable problem can be solved only numerically. A terse discussion of existing numerical approaches (explicit, implicit, ADI (Alternate Direction Implicit)) is presented with commentaries.

When a substance is passively transported by the moving fluid that contains the substance, such process is called advection (or passive scalar advection). Chapter 6 concentrates on describing this important and remarkable phenomenon through two examples: a dissolved species in a river and the motion of a lahar (the Indonesian term "lahar" refers to the rheological material, which contains a mixture of water and pyroclastic debris flowing down the slopes of a volcano). Four difference solution schemes to the (linear) advection equation are considered from the standpoint of the convergence and stability of these schemes, with the conclusion that the advection equation is rather difficult to solve accurately by finite difference.

In nature, transport processes occur in media through the combination of advection and diffusion. Chapter 7 incorporates diffusion into the advection equation and presents physical derivation of the 1-D transport equation with a source or sink for the case of constant diffusivity. Two geological examples are considered: transport of suspended sediment in a stream and sedimentary diagenesis in the context of the influence of burrows. Many computational schemes have been proposed for solving the advection–diffusion equation: a comparison of two numerical methods (QUICK and QUICKEST) is performed in the final section.

Chapter 8 treats the 1-D nonlinear transport problems. Perhaps, one of the most celebrated equations in applied mathematics is the (quasilinear) Burgers' equation. It describes the physical situations in which the viscous and nonlinear effects are equally important. After the short physical derivation of the problem, in particular, an exact solution to Burgers' equation is presented (without proof). It should be noted that this equation may have a very rich family of possible solutions! Next, the accuracy of three finite difference schemes FTCS explicit, FTCS implicit (FTCS denotes: forward-in-time, centered-in-space) and MacCormack, for solving Burgers' equation are discussed in frames of an analytic solution mentioned earlier. However, in order to obtain accurate numerical solutions of Burgers' equation, it is desirable to use high-order approximations in space and time. The remaining sections are devoted to physical derivation of the 1-D Reynolds' equation describing momentum transport in turbulent fluid flows.

In Chapter 9, the well posedness of the famous St. Venant's equations in a bounded domain, under basic regularity assumptions on this domain is considered. These equations describe gradually varied, unsteady, cross-sectionally averaged flow in a stream. Two representative computational schemes for such flow are presented and explained: an explicit scheme on a staggered grid, and a four-point implicit scheme on a regular grid. Next, the dam break problem upon a channel is sketched both experimentally and computationally. Let us notice that this problem has become an archetype in testing the nonlinear shallow water equations solvers (see, e.g., Dutykh D., Mitsotakis D., *On the relevance of the dam break problem in the context of nonlinear shallow water equations*. Discrete and Continuous Dynamical Systems, Ser. S, Vol. 3, No. 2, 2010, pp. 1–XX).

The last chapter, Chapter 10, concerns the description of some classes of 2-D fluid flows in the geophysical context. It provides the reader with basic knowledge in geophysical hydrodynamics and the minimal basis of the modern computational fluid dynamics. The set of equations describing 2-D vertically integrated flow in a basin is derived and used for the modeling of Lake Ontario wind-driven circulation. An explicit FTCS version scheme is used on an Arakawa "C" grid. It must be emphasised, however, that all hydrodynamical, turbulent models contain a number of dimensionless constants and empirical parameters. Most of them are estimated with a scarce degree of confidence.

The didactic value of this textbook is enhanced by a set of modeling exercises which accompany nine chapters of the book.

This textbook will surely be an invaluable book for graduate and advanced undergraduate students in geology, sedimentology and hydrology. The presentation is accessible to anyone who has a rudimentary acquaintance with elements of higher mathematics, general physics, chemistry, geology and elementary fluid mechanics. The text is written clearly and coherently, the illustrations and figures are of high quality, and the examples are chosen with great informative proficiency. I highly recommend the book also to lecturers as an introductory textbook to complex Earth systems modeling.